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Enhancing students' understanding of multivariate relationships through simulation-based learning: A quasi-experimental study

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Abstract

Interpreting how multiple variables interact is often challenging for students, especially when statistical learning goes beyond calculation to explanation. In many cases, students can apply procedures correctly but still struggle to make sense of relationships among variables. Although simulation tools have been introduced to support learning, less is known about how they contribute to students' understanding of multivariate relationships when combined with clear instructional design. This study explored the use of a simulation based learning approach with first year pharmacy students. A quasi experimental pre test–post test design was conducted with 85 students, together with an attitudinal survey involving 229 participants. Students in the experimental group worked with a simulation environment that allowed them to explore how variables interact under different conditions, while the control group followed conventional instruction. Students who engaged with the simulation achieved higher post test scores than those receiving conventional instruction ($t(83) = 2.096$, $p = 0.039$, $d = 0.46$). When pre test differences were taken into account, the instructional effect remained significant ($F(1,82) = 22.55$, $p < 0.001$, $\eta^2 = 0.216$). Students in the experimental group also reported greater engagement and confidence in interpreting statistical relationships. Students who worked with the simulation were better able to interpret how multiple variables interacted than those in the control group. From a practical standpoint, the study suggests that using simulation based environments in statistics instruction can support learning activities that foreground interpretation and conditional reasoning rather than routine procedural work, particularly in applied contexts such as pharmacy education.

Keywords: Conceptual understanding, Interpretive competence, Multivariate relationships, Realistic Mathematics Education, Simulation-based learning, Statistics education.

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1. Introduction

Interpreting relationships among multiple variables has become an important aspect of statistical learning in science and technology education. In applied fields such as pharmacy, students are expected not only to carry out calculations but also to explain how variables jointly influence outcomes and to interpret results in context. However, many students experience difficulty when they are required to move beyond procedures and make sense of relationships among variables.

This difficulty is particularly evident in multivariate situations, where several variables must be considered at the same time. Rather than viewing relationships as conditional, students often approach variables separately or focus on isolated numerical results. As a result, correct calculations do not necessarily reflect a clear understanding of how variables interact. This issue is especially relevant in pharmacy education, where data-informed reasoning plays an important role in formulation, optimisation, and quality evaluation. In such settings, understanding how multiple factors operate together is not simply a technical skill but part of a broader form of mathematical and statistical literacy needed for professional practice [1-3].

Among the models commonly used to represent these relationships, multivariate linear regression occupies an important place because it provides a way of expressing how several variables jointly influence an outcome [4, 5]. A general form of such a model can be written as

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_p x_p + \varepsilon,$$

where y denotes the outcome variable, $x_1, x_2, \dots, x_p$ represent predictor variables, and each coefficient β_j expresses the contribution of a predictor when the other variables are taken into account. From an educational perspective, the key issue is not the formulation itself, but whether students can interpret what each coefficient represents in relation to other variables. In other words, the challenge lies in helping students recognise regression as a way of reasoning about conditional relationships rather than as a set of isolated calculations [6, 7].

Research has consistently shown that students experience difficulties in developing conceptual understanding in statistics and related mathematical topics. Many can follow computational procedures successfully while still lacking a coherent understanding of what the results represent [8-10]. This problem becomes even more pronounced in multivariate situations, where students must coordinate several variables simultaneously and interpret how the effect of one variable depends on the presence of others. Under these conditions, students often fall back on simplified reasoning, treating variables independently or interpreting coefficients as if they referred to direct and isolated effects [11, 12]. What appears to be a correct numerical answer may therefore conceal a fragile or incomplete understanding.

The difficulty is compounded in applied disciplines such as pharmacy, where many first year students enter with limited mathematical confidence and often perceive statistics as abstract, technical, or disconnected from their main professional interests [13, 14]. Students' anxiety and low confidence in statistics can hinder meaningful learning and reduce their willingness to engage deeply with interpretation [13]. Although contextualisation has often been proposed as a way of addressing this problem, the use of context alone does not automatically produce deeper understanding. Students may recognise that a problem is relevant to their field yet still fail to connect contextual information with formal representations or mathematical meaning [15, 16].

For this reason, there is a need for instructional approaches that do more than simplify content or add contextual examples. What is required is a design that helps students actively build meaning for relationships among variables. Realistic Mathematics Education (RME) provides a relevant framework for addressing this issue by viewing mathematics as a human activity and emphasising the movement from informal understanding to more formal representations [17, 18]. Constructivist learning theory further supports this view by emphasizing that knowledge is not transmitted ready made, but constructed through interaction, reflection, and negotiation of meaning [19, 20]. Taken together, these perspectives suggest that students need opportunities to explore, test, discuss, and interpret relationships rather than only being shown how to calculate them.

Technology can support this process when it is used to enhance, rather than replace, learning [21]. In statistics and mathematics education, simulation based environments have been shown to help students engage with relationships dynamically, especially when those relationships are difficult to visualise through static examples alone [22-25]. By allowing learners to manipulate variables and immediately observe the consequences, such environments can make relationships more visible and open to interpretation. At the same time, this potential depends heavily on design. If the technological environment introduces too much technical complexity, it may distract students from the central conceptual issues. This is particularly relevant in contexts where programming is not itself a learning goal. In such cases, reducing unnecessary cognitive demands becomes essential if students are to focus on meaning rather than mechanics [26].

This study investigates a simulation-based instructional approach designed to support first-year pharmacy students in developing conceptual understanding of multivariate linear regression. The emphasis is placed not on teaching students to execute procedures more efficiently, but on helping them understand how variables relate, how outcomes change under different conditions and how model parameters can be interpreted in context. The learning environment was designed to support this process by enabling students to explore relationships dynamically without requiring programming skills.

Although conceptual understanding, simulation-based learning, and RME have been widely discussed in mathematics education, less is known about how these elements can be brought together to support students' understanding of relationships among multiple variables. In particular, there is still limited empirical evidence on how simulation-based environments, when combined with a clear instructional design, influence students' ability to interpret conditional relationships rather than simply carry out procedures.

This study addresses this issue by examining an instructional approach that integrates simulation-based learning with principles from RME and constructivist perspectives. The focus is on how students interpret relationships among variables and how their understanding develops through interaction with the learning environment.

Based on this perspective, the study addresses the following research questions:

RQ1: To what extent does a simulation-based instructional approach improve students' understanding of multivariate relationships, particularly in interpreting conditional effects?

RQ2: How does the instructional approach influence students' engagement, confidence, and perceived relevance of statistical learning?

RQ3: What is the magnitude of the instructional effect on learning outcomes after controlling for initial differences?

By addressing these questions, the study contributes to a better understanding of how simulation-based learning can support students' interpretation of relationships among variables in applied contexts.

2. Literature Review

2.1. Conceptual Understanding in Mathematics Education

Conceptual understanding refers to students' ability to understand relationships, structures, and meanings rather than merely apply procedures [27, 28]. In contexts involving data and variation, this includes the ability to interpret results, explain relationships among variables, and connect different forms of representation [29, 30].

A fundamental distinction is between procedural knowledge and conceptual understanding [31]. Procedural knowledge enables students to carry out calculations, whereas conceptual understanding allows them to explain why procedures work and how different quantities are related. In many educational settings, students develop procedural competence without corresponding conceptual insight, leading to difficulties in applying knowledge in new situations.

In multivariate contexts, conceptual understanding involves coordinating multiple variables and interpreting their relationships within a system. Students must recognise that the effect of one variable cannot be understood in isolation but depends on the presence of other variables. This requires a shift from linear, single-variable thinking to relational reasoning involving multiple interacting components.

Students often struggle with this type of understanding, focusing on numerical outputs without interpreting their meaning or recognising how variables jointly influence outcomes [29, 32]. These difficulties highlight the need for instructional approaches that explicitly support interpretation and reasoning, rather than focusing solely on computation.

In particular, developing interpretive competence, understood as the ability to explain relationships and make sense of model parameters, can be seen as a central aspect of conceptual understanding in multivariate contexts [29].

2.2. Learning Difficulties in Multivariate Contexts

Learning about relationships among multiple variables presents significant cognitive and conceptual challenges. Unlike simpler situations that can be easily visualised, multivariate relationships require students to consider several variables simultaneously and to understand how these variables interact.

One major difficulty lies in interpreting conditional relationships. Students often adopt intuitive interpretations in which variables are considered independently, without recognising that the effect of one variable depends on others. This leads to misconceptions, such as assuming that relationships are fixed or independent of context. Developing the ability to reason conditionally is a key challenge in multivariate learning, as it requires understanding how a change in one variable affects an outcome while other variables remain constant [33, 34].

Another challenge involves connecting representations. Students may encounter relationships through numerical data, graphs, and symbolic expressions, but they often struggle to move between these representations and to understand how they are related [32]. This lack of coordination limits their ability to interpret models and apply them meaningfully.

Cognitive load also plays an important role. Coordinating multiple variables and interpreting their relationships requires substantial mental effort. If instructional approaches introduce additional complexity, such as unfamiliar procedures or technical tools, students may focus on surface features rather than underlying relationships [26].

In applied disciplines such as pharmacy, these challenges are further intensified by students' limited mathematical background and their perception of statistics as abstract and disconnected from practice [13]. Therefore, effective instruction must address both conceptual and affective dimensions, supporting students not only in understanding relationships but also in developing confidence and engagement.

2.3. Realistic Mathematics Education and Constructivist Perspectives

RME (RME), developed by Freudenthal, frames mathematics learning as engagement in meaningful activity [17]. In this perspective, mathematics is viewed as a human activity in which learners construct knowledge by organising and structuring their experiences. Rather than presenting mathematical ideas as finished products, instruction should engage students in processes of exploration, reasoning, and reflection, a perspective that is also reflected in inquiry-based instructional models such as the 5E framework [35].

A central concept in RME is progressive mathematization, which describes how students move from informal reasoning grounded in context toward more formal mathematical representations [18]. This process involves the development of models that initially emerge from students' activity and gradually become more structured and generalised. Importantly, symbols and representations gain meaning through use, rather than being introduced as abstract entities.

In the context of learning relationships among variables, this perspective emphasises that students should first explore how variables co-vary in meaningful situations. Through guided activity, they begin to identify patterns, compare cases, and articulate relationships. These informal understandings can then be progressively formalised into more structured representations, supporting the development of conceptual understanding.

Constructivist learning theory emphasises that knowledge is actively constructed by learners [19, 20]. Learning occurs through interaction with tasks, reflection on experiences, and, in social constructivist views, collaboration with others. Understanding is not transmitted directly but develops through processes of interpretation and reorganisation.

In the context of multivariate relationships, this implies that students need to actively construct explanations of how variables interact, rather than relying on procedures that treat variables independently.

From this perspective, learning about multivariate relationships involves coordinating multiple ideas and constructing explanations that account for how variables interact. Students must move beyond observing patterns to explaining them, and from describing relationships to interpreting their meaning. This shift requires opportunities for active engagement, discussion, and reflection.

Both RME and constructivist perspectives highlight the importance of representations in learning. Students need to work with different forms of representation such as contextual, graphical, and symbolic forms, and to understand how these representations are connected. Developing this coordination is essential for understanding relationships among variables in a coherent way.

In this study, these theoretical perspectives inform the design of the instructional approach. The simulation-based environment is intended to support students' active construction of meaning by allowing them to explore relationships dynamically, test ideas, and reflect on outcomes. Through guided interaction, students are encouraged to interpret relationships, explain their reasoning, and connect different representations.

2.4. Simulation-Based Learning and Representation

Simulation-based learning environments are used to support conceptual understanding in mathematics education [22, 23, 31]. By allowing students to manipulate variables and observe resulting changes, simulations make relationships more visible and accessible.

In multivariate contexts, simulation plays a particularly important role because relationships are often difficult to visualise directly. By interacting with a simulation, students can explore how changes in one variable affect outcomes while other variables remain unchanged. This supports the development of conditional reasoning and helps students understand how variables interact within a system.

Simulation environments also facilitate the coordination of multiple representations. Students can observe relationships through tables, graphs, and symbolic expressions simultaneously, which supports connections between different forms of representation [1]. This coordination is essential for developing conceptual understanding and for transferring knowledge to new contexts.

However, the effectiveness of simulation depends on its design. If the environment is overly complex or requires additional technical skills, such as programming, students may become overwhelmed. This can increase cognitive load and reduce the effectiveness of learning [26]. Therefore, simulation tools should be designed to support exploration and interpretation without introducing unnecessary barriers.

In addition, simulation-based learning must be integrated with appropriate instructional guidance. Simply interacting with a simulation is not sufficient; students need support in interpreting results, articulating relationships, and reflecting on their understanding. Guided activities and structured tasks play an important role in ensuring that simulation leads to meaningful learning.

In this sense, simulation does not function merely as a technological tool, but as a means of supporting the development of conceptual understanding through interaction, representation, and interpretation.

2.5. Research Gap and Contribution of the Study

Although research exists on conceptual understanding, RME and simulation-based learning, less is known about how these elements can be integrated in teaching relationships among multiple variables. Many existing approaches either focus on procedural aspects or use technology without a clear pedagogical framework.

In particular, there is limited empirical evidence on how simulation-based environments grounded in educational theory can support students' ability to interpret relationships among variables and to explain the meaning of model parameters. Few studies have examined not only overall learning outcomes but also specific aspects of understanding, such as interpretive competence and the ability to articulate relationships.

This study addresses this gap by combining a theory-driven instructional approach with a simulation-based learning environment. The focus is on how students develop understanding of relationships among variables, how they interpret these relationships in context and how their attitudes toward learning are affected.

The study contributes to mathematics education in science and technology by showing how technology-supported instruction can support reasoning, interpretation and engagement in learning about relationships among variables.

Supporting students' conceptual understanding of multivariate relationships requires drawing on multiple theoretical perspectives. Conceptual understanding serves as the learning goal, whereas RME and constructivist perspectives inform the design of meaningful learning processes. Simulation-based environments provide a way to make relationships visible and open to exploration. Building on this perspective, the present study examines how these elements can be combined to support students' interpretation of relationships among variables.

3. Methodology

3.1. Research Design

This study adopted a quasi-experimental pre-test–post-test control group design to examine a simulation-based instructional approach and its impact on students' understanding of relationships among multiple variables. The design allows comparison between instructional approaches in authentic classroom settings, where random assignment is often not feasible [8, 32].

Two groups of students participated in the study. The two groups were formed based on intact class assignments, as random allocation was not feasible in the institutional context. Both groups were taught by the same instructor to minimise instructional variability. The intervention was implemented over 2 weeks, with equivalent instructional time allocated to both groups.

The experimental group was exposed to a simulation-based learning environment designed to support exploration and interpretation of relationships among variables, while the control group received conventional instruction focusing primarily on procedures and calculations. Both groups were assessed before and after the intervention to capture changes in students' understanding.

The use of a pre-test–post-test structure served two important purposes. First, it enabled the identification of baseline equivalence between groups, ensuring that any observed differences in post-test performance could be attributed more confidently to the instructional intervention. Second, it allowed for the measurement of learning gains over time, which is particularly important when investigating conceptual understanding rather than immediate performance.

In addition to quantitative measures, the study incorporated qualitative data to provide a more comprehensive account of students' learning processes. Students' written responses and classroom interactions were analysed to examine how they interpreted relationships among variables and how their reasoning evolved during the instructional sequence. This mixed-methods approach aligns with research in mathematics education [10] where understanding is examined through both measurable outcomes and students' explanations and representations [23, 29].

The design reflects the study's theoretical framework, emphasising active learning and meaning construction. Comparing two instructional approaches within the same context allows the study to explore both the effectiveness of the intervention and its impact on how students understand relationships among variables.

3.2. Participants and Context

The study was conducted with first-year undergraduate students enrolled in a pharmacy programme. These students were taking a statistics-related course in which they were introduced to concepts involving data analysis and relationships among variables. A total of $n = 85$ students participated in the quasi-experimental component of the study, with students divided into an experimental group and a control group based on their existing class assignments.

Students in applied disciplines such as pharmacy are often required to work with data involving multiple variables, yet they may not have a strong mathematical background. As a result, they tend to approach statistical concepts procedurally, focusing on calculations rather than interpretation [29]. This makes the development of conceptual understanding especially challenging.

To complement the analysis of learning outcomes, an attitudinal survey was administered to a larger sample of $n = 229$ students. This broader dataset provided insight into students' perceptions of statistics, their level of engagement, and their confidence in interpreting relationships among variables. Including a larger sample for attitudinal analysis strengthens the generalisability of findings related to affective factors.

The participants had limited prior experience with multivariate reasoning and were more familiar with single-variable or bivariate situations. This background influenced how they approached learning tasks and highlights the importance of instructional support in helping students transition from simple to more complex forms of reasoning.

The instructional intervention was implemented within regular classroom settings, ensuring ecological validity. This means that the findings of the study are directly relevant to real educational practice, rather than being limited to controlled experimental conditions.

3.3. Instructional Design and Learning Environment

The instructional design was grounded in RME and constructivist learning theory, focusing on supporting students' understanding of relationships among multiple variables through meaningful activity. Rather than presenting multivariate models as formal procedures, the intervention was designed to help students construct meaning by exploring how variables interact within realistic contexts.

Central to the intervention was a simulation-based learning environment that allowed students to manipulate input variables and observe corresponding changes in outcomes. This environment was specifically designed to support the development of conditional reasoning. It allowed students to examine how a change in one variable influences the outcome while other variables are held constant. Such functionality is essential for understanding multivariate relationships, where the effect of a variable cannot be interpreted in isolation.

The instructional sequence followed a progression from contextual exploration to more structured representation, consistent with the principle of progressive mathematization [18]. Students were first introduced to contextual problems relevant to their field of study, such as situations involving multiple factors influencing a pharmaceutical outcome. Through guided exploration, they were encouraged to identify patterns, compare cases, and describe relationships among variables using informal language.

As students engaged with the simulation, they were prompted to articulate how changes in variables affected outcomes and to justify their reasoning. This process was supported by tasks requiring explanation, comparison, and interpretation rather than computation. Over time, these informal understandings were gradually connected to more formal representations, allowing students to interpret model parameters as meaningful quantities rather than abstract symbols.

An important design consideration was the reduction of cognitive load. The simulation environment was developed to be accessible without requiring programming skills, thereby allowing students to focus on conceptual understanding rather than technical implementation. Excessive technical demands can hinder learning when the goal is to support interpretation and reasoning [26].

In addition, classroom interactions played a key role in the instructional process. Students were encouraged to discuss their ideas, compare interpretations, and reflect on differences in reasoning. These interactions supported the social construction of meaning, as students negotiated and refined their understanding of relationships among variables.

The instructional design also incorporated opportunities for students to engage with multiple representations, including contextual descriptions, numerical outputs, and graphical displays. By coordinating these representations, students were able to develop a more integrated understanding of multivariate relationships. This aspect of the design is particularly important, as the ability to move between representations is a key component of conceptual understanding in mathematics education [1].

In this approach, the instructional design shifted the focus from performing calculations to understanding underlying relationships. Through activities involving exploration, interpretation, and discussion, students were encouraged to build knowledge that is both meaningful and applicable in new contexts.

3.4. Instruments

To capture different aspects of students' learning, the study employed a set of instruments designed to assess conceptual understanding, interpretive competence, and attitudes toward learning. These instruments were closely aligned with the objectives of the instructional intervention and the theoretical framework of the study.

The primary instrument for measuring learning outcomes was a conceptual understanding test administered before and after the intervention. The test was designed to assess students' ability to interpret relationships among variables rather than their ability to perform calculations. Items required students to explain how variables are related, describe how changes in one variable affect outcomes, and interpret the meaning of model parameters in context. This emphasis on explanation and interpretation reflects the view that conceptual understanding involves making sense of relationships rather than applying procedures [29].

In addition to overall test scores, the assessment was structured into components reflecting key aspects of understanding. These components, derived from the design of the assessment instrument, included understanding relationships among variables, constructing representations, and interpreting parameters. By analysing these components separately, the study was able to identify specific areas in which students demonstrated improvement. This approach provides a more nuanced understanding of learning outcomes and allows for the identification of strengths and weaknesses in students' conceptual development.

The design of the test items was informed by previous research in statistics education, which emphasises the importance of interpretation and reasoning [30, 32]. Items were contextualised to reflect real-world situations relevant to students' field of study, thereby supporting engagement and relevance.

To examine students' attitudes toward learning, an attitudinal survey was administered to a larger sample of n = 229 students. The survey included Likert-scale items measuring interest in statistics, confidence in understanding relationships among variables, and perceived relevance of the content. These dimensions are important in mathematics education, as affective factors can influence both engagement and learning outcomes.

Qualitative data were also collected to complement the quantitative measures. Students' written responses to test items were analysed to examine how they articulated relationships and how their reasoning developed. In addition, classroom observations provided insight into how students interacted with the simulation environment and how they engaged with instructional activities.

The data collected from these instruments allowed for the examination of both learning outcomes and processes. The use of both quantitative and qualitative data reflects common practice in mathematics education research, where understanding is seen as a complex construct that cannot be fully represented by numerical measures alone [23].

3.5. Data Analysis

Data were analysed to examine overall learning outcomes and specific aspects of conceptual understanding. Statistical analyses were complemented by qualitative analysis to provide a comprehensive account of students' learning.

Prior to conducting parametric analyses, assumptions of normality and homogeneity of variance were examined. The Shapiro–Wilk test indicated no significant deviations from normality, and Levene's test confirmed homogeneity of variance ($p > 0.05$). For ANCOVA, the assumption of homogeneity of regression slopes was also tested and satisfied.

To establish baseline equivalence between groups, pre-test scores were analysed using an independent-samples t-test. The results indicated no statistically significant difference between the experimental and control groups ($p > 0.05$), suggesting that the two groups were comparable prior to the intervention.

To evaluate the impact of the instructional approach on learning outcomes, post-test scores were compared between groups using independent-samples t-tests. In addition to statistical significance, effect size was calculated using Cohen's d

to assess the magnitude of the observed differences. Reporting effect size is particularly important in educational research, as it provides information about the practical significance of findings.

To obtain a more precise estimate of the instructional effect, an analysis of covariance (ANCOVA) was conducted with pre-test scores as the covariate and post-test scores as the dependent variable. The results were evaluated using the F-statistic and partial eta squared η^2, which indicates the proportion of variance in post-test scores explained by the instructional approach. This analysis controls for initial differences and estimates the effect of the intervention more precisely [25].

Component-based analysis was conducted to examine how the instructional approach influenced different aspects of conceptual understanding. Scores corresponding to each component were analysed separately, allowing for the identification of specific areas of improvement, particularly in relation to interpretive competence.

Survey data were analysed using descriptive and inferential statistics to compare attitudes between groups. Differences in mean scores were examined to assess the impact of the instructional approach on students' engagement and confidence.

Qualitative data were analysed thematically to identify patterns in students' reasoning and interpretation. Particular attention was given to how students described relationships among variables, how they explained changes in outcomes and how their reasoning evolved during the instructional process. These qualitative findings were used to support and enrich the interpretation of quantitative results.

Regarding the analytical framework, the data analysis strategy was designed to align with the objectives of the study and to provide a comprehensive understanding of how the instructional approach influenced both learning outcomes and students' ways of thinking about relationships among variables.

4. Results

Pre-test scores showed no statistically significant difference between the experimental group (D25A) and the control group (D25C) prior to the intervention. As presented in Table 1, the mean pre-test score of the experimental group was 5.68 ± 1.42, while that of the control group was 5.96 ± 1.41. The independent-samples t-test yielded $t(83) = -0.905, p = 0.368$, and the two groups were comparable at baseline.

Following the intervention, a statistically significant difference was observed in post-test scores. The experimental group achieved a higher mean score (6.26 ± 1.38) compared to the control group (5.68 ± 1.36), with $t(83) = 2.096, p = 0.039$. The effect size was d = 0.46, indicating a moderate impact of the instructional approach [9]. Post-test scores were higher in the experimental group than in the control group.

Table 1.
Comparison of pre-test and post-test scores.

Variable	**D25A ($n = 40$)** $M \pm SD$	**D25C ($n = 45$)** $M \pm SD$	$t(df)$	p	**Cohen's d**
Pre-test	5.68 ± 1.42	5.96 ± 1.41	−0.905 (83)	0.368	−0.20
Post-test	6.26 ± 1.38	5.68 ± 1.36	2.096 (83)	0.039	0.46

To further examine the effect of the instructional approach while controlling for pre-test scores, an analysis of covariance was conducted. As shown in Table 2, the instructional condition had a statistically significant effect on post-test performance ($F(1,82) = 22.55, p < 0.001$), with a partial effect size of $\eta^2 = 0.216$. The instructional condition accounted for a meaningful proportion of the variance in post-test scores, as reflected in the partial η^2 value.

Table 2.
ANCOVA results.

Source	$F(1, 82)$	p	**Partial η^2**
Instructional condition	22.55	< 0.001	0.216

The adjusted means presented in Table 3 show the difference between groups after controlling for initial performance. The experimental group achieved an adjusted mean of 6.392, compared to 5.612 for the control group, with a mean difference of 0.780 (95% CI: [0.453, 1.107]). The adjusted means show a consistent advantage for the experimental group after controlling for initial performance.

Table 3.
Adjusted means from ANCOVA.

Group	**Adjusted Mean**	**95% CI**
Intervention	6.392	[6.155, 6.629]
Control	5.612	[5.388, 5.835]
Mean difference	0.780	[0.453, 1.107]

Beyond overall performance, differences between groups were particularly evident in tasks requiring interpretation of relationships among variables. Students in the experimental group were more likely to explain how changes in one variable affected outcomes while considering the role of other variables. In contrast, students in the control group often focused on individual values or procedural steps without clearly articulating relationships. This pattern reflects differences in how students engaged with relationships, compared to a more procedural focus. This is evident in how relationships among variables were interpreted, not only in score differences.

The role of the simulation environment in supporting this development is illustrated in Figure 1. The environment enabled students to manipulate variables dynamically and observe corresponding changes in outcomes, making relationships more visible and accessible.

Figure 1.
Simulation output illustrating dynamic relationships among variables.

This visualisation highlights how changes in individual variables lead to different outcomes depending on the values of other variables, reinforcing the idea of conditional relationships.

The attitudinal data presented in Table 4 show that students in the experimental group reported more positive perceptions of learning. Significant differences were observed in affect, cognitive competence, perceived difficulty, interest and effort. For example, the difference in affect was +0.47 ($p < 0.001$), and in cognitive competence was +0.38 ($p < 0.001$). These results correspond to RQ2 and show higher levels of engagement, confidence and more positive attitudes toward learning in the experimental group.

Table 4.
Attitudinal dimensions.

Facet	D25A ($N = 115$) Mean (SD)	D25C ($N = 114$) Mean (SD)	Difference	p(U-test)
Affect	3.65 (0.62)	3.18 (0.70)	+0.47	< 0.001*
Cognitive Competence	3.72 (0.55)	3.34 (0.65)	+0.38	< 0.001*
Value	3.92 (0.50)	3.80 (0.58)	+0.12	0.082
Difficulty	3.08 (0.58)	2.65 (0.61)	+0.43	< 0.001*
Interest	3.48 (0.75)	3.05 (0.81)	+0.43	< 0.001*
Effort	3.78 (0.59)	3.55 (0.66)	+0.23	0.015*

More detailed survey results presented in Table 5 further support these findings. A higher proportion of students in the experimental group reported improvements in understanding, application and interpretation. In particular, 68% of students indicated that they were able to interpret regression results, compared to 48% in the control group. This difference can be attributed to the contribution of the instructional approach to students' meaningful understanding.

Table 5.
Selected survey items.

Question (Excerpt)	D25A (Agree %)	D25C (Agree %)	Professional Meaning
Q7: Understanding basic statistical concepts	78%	62%	Conceptual understanding
Q10: Sufficient skills for real-world application	72%	55%	Application
Q12: Ability to interpret regression results	68%	48%	Interpretation

Additional indicators summarised in Table 6 show that students in the experimental group experienced higher engagement, lower anxiety and stronger perceived relevance of the content. These factors are important for sustaining learning and supporting the application of knowledge in professional contexts.

Table 6.
Additional indicators.

Indicator Group	Key Variables	Notable Observed Results
Engagement	Q32	Highest mean = 4.25
Confidence	Q2, Q4, Q6	Lower anxiety
Relevance	Q15	Better understanding of medical research

The development of students' reasoning is further illustrated in Figure 2. Students in the experimental group demonstrated a clearer representation of relationships among variables, reflecting an understanding of how variables interact within a system rather than being treated independently.

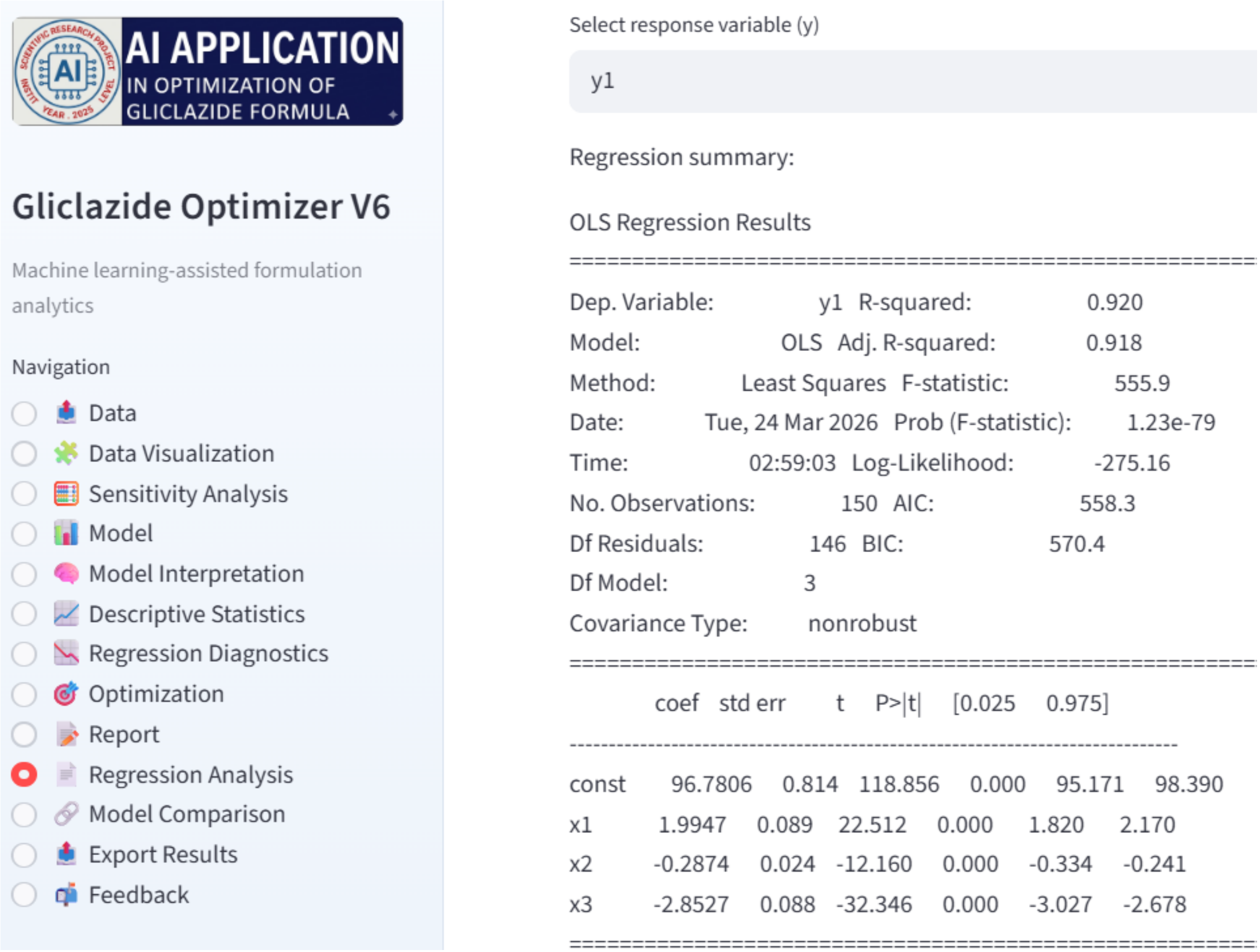

Figure 2.
Example of students' representations showing relational understanding of variables.

The representation indicates that students in the experimental group were able to coordinate multiple variables simultaneously, rather than interpreting them in isolation. The results demonstrate improvements not only in performance but also in students' ability to interpret conditional relationships, reflecting a shift from procedural to relational reasoning. The approach was also associated with higher levels of engagement and confidence, as well as a measurable instructional effect.

5. Discussion

A clear difference emerged in how students approached multivariate relationships. Students in the simulation-based group shifted from treating variables independently to interpreting how variables interact within a system.

This shift is especially important in multivariate thinking, where students often begin with direct and simplified interpretations of variable effects. Prior studies have shown that learners frequently assume each variable operates independently [11, 12]. The simulation-based environment challenged this assumption by making conditional relationships explicit. When students observed that the effect of one variable changed depending on the values of others, they were prompted to reconsider how multivariate relationships operate.

In RME, this development represents a shift from informal, experience-based reasoning toward more structured understanding [17, 18]. Rather than introducing relationships as fixed objects, the instructional design enabled students to encounter them through activity and observation. Interaction with the simulation supported this progression by helping students connect observable behaviour with increasingly formal interpretations.

From a constructivist perspective, learning emerges through interaction and reflection, both in individual knowledge construction and classroom-based processes [6, 36]. The simulation environment required students to interpret outcomes rather than simply follow procedures. When results did not match expectations, students revised their reasoning and adjusted their interpretations. This process may have supported deeper learning and helped students move beyond procedural thinking. In contrast, students in the control group had fewer opportunities to engage in such reflective activity, and their responses remained more procedural.

A clear difference between the two groups is evident in their ability to interpret relationships. Students in the experimental group were more likely to explain how variables influence one another and to recognise that these relationships depend on context. This form of interpretive competence is important for working with data in applied settings and aligns with research on statistical reasoning that emphasises interpretation over procedural calculation [16, 29]. It is also an aspect of learning that is often underdeveloped when instruction focuses primarily on obtaining correct answers rather than understanding their meaning.

Visualisation played an important role in supporting this development. The simulation made relationships visible and allowed students to explore them dynamically. The coordination of multiple representations supports conceptual understanding when learners relate them meaningfully [1] a finding also supported in research on statistical learning [16, 29]. In this study, the integration of these representations supported more coherent interpretations of relationships among variables.

From a teaching perspective, the importance of organising instruction around exploration rather than formulas alone becomes clear. In multivariate contexts, allowing students to manipulate variables and observe how outcomes change may support a more coherent understanding of how statistical models represent real-world situations.

Changes in students' attitudes reinforce this interpretation. Increased confidence and interest were associated with the way students experienced the learning process. As they explored relationships and observed the consequences of their actions, the content became more accessible and meaningful. This points to a close relationship between engagement and the structure of learning environments, particularly in subjects often perceived as abstract or difficult [13].

For teaching in applied disciplines such as pharmacy, the results draw attention to several important considerations. Students are expected to interpret data and understand relationships, yet instruction often emphasises techniques over meaning. Providing opportunities for exploration and interpretation may support the development of more meaningful forms of understanding. Engaging students with relationships through interaction may be more effective than introducing complexity through formal procedures at an early stage.

While the study is limited to a specific context, including the participants, tasks and instructional design, it provides insight into how students develop understanding of relationships among variables through guided interaction. The relatively short duration of the intervention also suggests the need for further research examining how such approaches can be sustained over longer periods.

The study focuses on how students develop understanding of relationships rather than only their ability to compute them. The simulation-based approach supported this process by making relationships visible, encouraging exploration and requiring interpretation. Its value lies in improving outcomes while shaping how students engage with and interpret mathematical ideas.

6. Conclusion

This study examined how students understand relationships among multiple variables in situations where procedural work often dominates. The findings suggest a change in how students approached these relationships. Rather than treating variables as separate elements, many began to recognise that outcomes depend on how variables operate together.

Working with the simulation may have supported this shift by placing students in situations where relationships had to be interpreted. When values changed in ways that did not match initial expectations, students were pushed to reconsider their reasoning and look for explanations that accounted for more than one factor. This process encouraged a form of thinking in which relationships are seen as conditional rather than fixed.

This shift is particularly valuable in contexts where students must work with data beyond the classroom. The ability to interpret relationships is essential in evidence-based fields, and the findings suggest that learning environments built on interaction and interpretation help develop forms of reasoning that transfer effectively to such settings.

The simulation environment played a supporting role in this process. It made relationships visible and open to exploration. Students were able to test ideas, observe patterns and adjust their understanding as they worked. This created conditions in which meaning could be built gradually, rather than imposed through formal expressions from the outset.

The findings are specific to the participants, tasks and learning environment in this study, which together contributed to the observed outcomes. The patterns that emerged indicate the value of approaches that allow students to experience how relationships behave before formalising them.

This study highlights the value of instructional approaches that prioritise interpretation and relational reasoning over procedural performance. When attention is given to how students make sense of relationships, rather than only how they compute them, different forms of understanding can develop that are more useful in contexts where interpretation is required.

References

[1] S. Ainsworth, "DeFT: A conceptual framework for considering learning with multiple representations," *Learning and Instruction,* vol. 16, no. 3, pp. 183-198, 2006. https://doi.org/10.1016/j.learninstruc.2006.03.001

[2] S. Bolton and C. Bon, *Pharmaceutical statistics: Practical and clinical applications*. London, U.K: Informa Healthcare, 2009.

[3] D. Ben-Zvi, "Toward understanding the role of technological tools in statistical learning," *Mathematical Thinking and Learning,* vol. 2, no. 1-2, pp. 127-155, 2000. https://doi.org/10.1207/S15327833MTL0202_6

[4] R. W. Bybee *et al.*, *The BSCS 5E instructional model: Origins and effectiveness*. Colorado Springs, CO, USA: BSCS, 2006.

[5] B. Chance, R. delMas, and J. Garfield, *Reasoning about sampling distributions," in the challenge of developing statistical literacy, reasoning, and thinkinga, D. Ben-Zvi and J. Garfield, Eds*. Dordrecht, Netherlands: Kluwer, 2004.

[6] P. Cobb, "Where is the mind? Constructivist and sociocultural perspectives on mathematical development," *Educational Researcher*, vol. 23, no. 7, pp. 13-20, 1994. https://doi.org/10.3102/0013189X023007013
[7] P. Cobb, "Individual and collective mathematical development: The case of statistical data analysis," *Mathematical Thinking and Learning*, vol. 1, no. 1, pp. 5-43, 1999. https://doi.org/10.1207/s15327833mtl0101_1
[8] L. Cohen, L. Manion, and K. Morrison, *Research methods in education*. London, U.K: Routledge, 2018.
[9] J. Cohen, *Statistical power analysis for the behavioral sciences*. Hillsdale, NJ, USA: Lawrence Erlbaum, 1988.
[10] J. W. Creswell and V. L. Plano Clark, *Designing and conducting mixed methods researchc*. Thousand Oaks, CA, USA: SAGE, 2018.
[11] T. De Jong and W. R. Van Joolingen, "Scientific discovery learning with computer simulations of conceptual domains," *Review of Educational Research*, vol. 68, no. 2, pp. 179-201, 1998. https://doi.org/10.3102/00346543068002179
[12] J. Fox, *Applied regression analysis and generalized linear models*. Thousand Oaks, CA, USA: SAGE, 2016.
[13] J. V. Freeman, S. Collier, D. Staniforth, and K. J. Smith, "Innovations in curriculum design: A multi-disciplinary approach to teaching statistics to undergraduate medical students," *BMC Medical Education*, vol. 8, no. 1, p. 28, 2008. https://doi.org/10.1186/1472-6920-8-28
[14] H. Freudenthal, *Revisiting mathematics education: China lectures*. Dordrecht, Netherlands: Kluwer, 1991.
[15] J. Garfield and D. Ben-Zvi, *Developing students' statistical reasoning: Connecting research and teaching practice*. New York, NY, USA: Springer, 2008.
[16] J. Garfield and D. Ben-Zvi, "Helping students develop statistical reasoning: Implementing a statistical reasoning learning environment," *Teaching Statistics*, vol. 31, no. 3, pp. 72-77, 2009. https://doi.org/10.1111/j.1467-9639.2009.00363.x
[17] K. Gravemeijer, *Developing realistic mathematics education*. Utrecht, Netherlands: CDβ Press, 1994.
[18] K. Gravemeijer and M. Doorman, "Context problems in realistic mathematics education: A calculus course as an example," *Educational studies in mathematics*, vol. 39, no. 1, pp. 111-129, 1999. https://doi.org/10.1023/A:1003749919816
[19] J. Hiebert and P. Lefevre, *Conceptual and procedural knowledge in mathematics: An introductory analysis, in Conceptual and Procedural Knowledge: The Case of Mathematics, J. Hiebert, Ed*. Hillsdale, NJ, USA: Lawrence Erlbaum, 1986.
[20] International Council for Harmonisation, *Pharmaceutical development Q8(R2)*. Geneva, Switzerland: ICH, 2009.
[21] J. Kilpatrick, J. Swafford, and B. Findell, *Adding it up: Helping children learn mathematics*. Washington, DC, USA: National Academy Press, 2001.
[22] M. H. Kutner, *Applied linear statistical models*, 5th ed. Boston, MA, USA: McGraw-Hill, 2005.
[23] S. J. Lane-Getaz, *What is statistical literacy? in Thinking and Reasoning with Data and Chance, G. Burrill, Ed*. Reston, VA, USA: NCTM, 2007.
[24] C. Mallows, "The zeroth problem," *The American Statistician*, vol. 52, no. 1, pp. 1-9, 1998.
[25] D. C. Montgomery, *Design and analysis of experiments*, 9th ed. Hoboken, NJ, USA: Wiley, 2017.
[26] D. S. Moore, "New pedagogy and new content: The case of statistics," *International Statistical Review*, vol. 65, no. 2, pp. 123-137, 1997. https://doi.org/10.1111/j.1751-5823.1997.tb00390.x
[27] D. L. Neumann, M. Hood, and M. M. Neumann, "Using real-life data when teaching statistics," *Statistics Education Research Journal*, vol. 12, no. 2, pp. 59–70, 2013.
[28] A. J. Onwuegbuzie and V. A. Wilson, "Statistics Anxiety: Nature, etiology, antecedents, effects, and treatments--a comprehensive review of the literature," *Teaching in higher education*, vol. 8, no. 2, pp. 195-209, 2003. https://doi.org/10.1080/1356251032000052447
[29] R. D. Pea, *Cognitive technologies for mathematics education, in Cognitive Science and Mathematics Education, A. H. Schoenfeld, Ed*. Hillsdale, NJ, USA: Lawrence Erlbaum, 1987.
[30] J. Piaget, *The principles of genetic epistemology*. New York, USA: Basic Books, 1972.
[31] N. Rutten, W. R. Van Joolingen, and J. T. Van Der Veen, "The learning effects of computer simulations in science education," *Computers & Education*, vol. 58, no. 1, pp. 136-153, 2012. https://doi.org/10.1016/j.compedu.2011.07.017
[32] W. R. Shadish, T. D. Cook, and D. T. Campbell, *Experimental and quasi-experimental designs for generalized causal inference*. Boston, MA, USA: Houghton Mifflin, 2002.
[33] S. Sharma, "Definitions and models of statistical literacy: a literature review," *Open Review of Educational Research*, vol. 4, no. 1, pp. 118-133, 2017. https://doi.org/10.1080/23265507.2017.135431
[34] R. R. Skemp, "Relational understanding and instrumental understanding," *Mathematics teaching*, vol. 77, no. 1, pp. 20-26, 1976.
[35] R. D. Snee, "What's missing in statistical education?," *The american statistician*, vol. 47, no. 2, pp. 149-154, 1993. https://doi.org/10.1080/00031305.1993.10475964
[36] E. von Glasersfeld, *Radical constructivism: A way of knowing and learning*. London, U.K: Falmer Press, 1995.