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Noise source localization based on spatial detector arrays

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Abstract

The purpose of this study is to investigate the applicability of a recently proposed method for wave source localization based on the Time Difference of Arrival (TDOA) and to reduce the problem to a set of linear equations. We apply TDOA to the noise source localization problem in urban environments. The study employs mathematical modeling using the three-dimensional acoustic wave equation for a point source. Simulations are performed with real noise samples recorded near a construction site. Various configurations of source positions relative to the detector array are considered. The localization accuracy is analyzed as a function of the geometric parameters of the detector array and the duration of the recorded signal segment. After determining the source coordinates, the signal amplitude in the vicinity of the source is reconstructed. Our findings indicate that the highest localization accuracy and signal reconstruction quality are achieved when the distances between the sensors are comparable to the distance to the source. The results demonstrate that, despite measurement uncertainties, the modeled approach remains robust. These insights have practical implications for the development of real-time acoustic monitoring systems in urban and industrial settings, such as noise source detection at construction sites or for public safety applications.

Keywords: Acoustics, Inverse source problem, Point source, Sound localization, Time difference of arrival, Urban noise, Wave equation.

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1. Introduction

Weak acoustic, electromagnetic, and elastic waves in a homogeneous medium obey the same 3D wave equation. This reflects the common mathematical structure of these physical phenomena [1]. This commonality allows for the development of universal modeling approaches applicable across various domains, including radio wave source localization, urban noise detection, indoor sound recognition, and drone positioning. Accurate source localization is crucial for monitoring and control in industrial and urban settings.

In many real-world scenarios, such as those involving aircraft, drones, or mobile sound sources, the object emitting the signal may be in motion. If the source speed is much lower than the wave speed, and the time window is short, the source can be treated as stationary. This simplifies localization and makes it tractable using wave-based mathematical methods.

This study focuses on the mathematical modeling and computational implementation of such localization techniques under realistic conditions. In particular, we explore the performance and limitations of a recently proposed time-delay-based method [2] for acoustic source localization in urban environments. The method is adapted and tested using real noise recordings collected near a construction site. The goal is to evaluate the robustness and practical applicability of the method for real-time detection and reconstruction of acoustic sources in urban settings. Our main contribution is testing the method from [2] using a hybrid model that combines acoustic wave simulations with real noise recordings. Synthetic data are generated under controlled variations in source position, detector geometry, and signal duration.

The paper is structured as follows: Section 2 provides a literature review; Section 3 (Methodology) sequentially presents the mathematical model used for simulating detected signals, details of data collection, a description of the sensor system, and the derivation of a closed-form linear system for computing source coordinates; Section 4 presents the results and discussion; finally, the conclusion summarizes the key findings and outlines potential directions for future research.

2. Literature Review

The problem of identifying an unknown source, known as the inverse source problem (ISP), is typically formulated within the framework of inverse problem theory [3] for the wave equation. However, a variety of engineering approaches exist that rely on assumptions about the relative positions of sources and detectors, as well as the nature of the emitted waves. For example, at large distances, incoming waves can be approximated as planar, which enables the estimation of the number of sources using the rank of a matrix constructed from the detected signal vectors [4-6]. This technique is widely known as the Multiple Signal Classification (MUSIC) method.

A limitation of the MUSIC method in practical applications is its high sensitivity to noise. Radio wave source localization for aerial vehicles has been explored in [4, 7] while the problem of determining the number of sound sources in indoor environments has been addressed in [5, 6, 8-10]. For identifying multiple simultaneously active sources, the Time Difference of Arrival (TDOA) method has also been employed, as seen in [5, 6, 8-11].

Other widely used techniques include the Maximum Likelihood (ML) method [12-14] and the beamforming method (BM), which utilizes signal delays between pairs of closely spaced sensors to estimate the direction of the source in space [15-18]. Comprehensive discussions on sound source identification are available in the book [19] and the review article [20], which also classifies various sound localization methods. Notably, the review in Liaquat et al. [20] developed with the assistance of AI techniques it provides an extensive list of relevant references in this field.

Machine learning methods are increasingly employed in sound analysis and recognition tasks. The survey in Grumiaux et al. [21] offers a structured overview of deep learning approaches to sound source localization (SSL). In He et al. [22], neural networks are applied to jointly analyze data from microphone arrays and cameras for source localization. A broader review covering 38 studies is presented in Desai and Mehendale [23], summarizing SSL techniques based on multiaural and binaural signals and combining conventional algorithms with convolutional neural networks (CNNs). This review categorizes SSL systems by the number of microphones, array configurations, localization algorithms, and 3D spatial capabilities.

The survey Jekateryńczuk and Piotrowski [24] emphasizes that classic physical-model methods, including Time Difference of Arrival (TDOA), remain essential for real-world applications, highlighting their broad utility in urban signal detection and monitoring. This review cites several recent papers in which the TDOA method has demonstrated good accuracy in determining the distance to a source for various civil applications. As Mensing and Plass [25] notes, the classical TDOA method suffers from typical issues of nonlinear iterative solvers: slow or absent convergence, high sensitivity to initial guesses, and multiple possible solutions. To overcome these challenges, various improvements to iterative methods have been developed in recent years, such as the weighted least squares method [26] and the combined TDOA with Frequency Difference Of Arrival (FDOA) method [27] (the latter is applicable in scenarios involving moving targets and significant Doppler effect influence).

In our study, we focus on identifying noise sources in urban environments, specifically at construction sites. Unlike the speech- or music-based scenarios discussed in Schmidt [4], our attention is on non-speech, non-musical, and non-RF (radio frequency) signals essentially, environmental noise. As a result, traditional metrics like the signal-to-noise ratio (SNR) are not applicable.

To address this, we adopt a TDOA-based method using closed-form equations for source localization, as introduced in the seminal work [2]. The approach reformulates the spatial localization problem as a linear system of equations, in contrast to earlier methods that used analytical TDOA-based formulas, if possible, or applied an iterative process.

Our previous work, Mukanova et al. [18] considered localization under the assumption that sources and detectors lie in the same plane. In this study, we extend the analysis to full 3D localization and focus on signal reconstruction from a single source using a spatially distributed microphone array.

3. Methodology

This study employs a physics-based acoustic wave model for localizing a sound source in a three-dimensional environment. The method is grounded in the classical formulation of the acoustic wave equation for a point source in a homogeneous medium and relies on calculating time delays for various configurations of the detector–source system.

The classical TDOA localization method is based on solving a system of nonlinear equations of the form "$\|\boldsymbol{r}_s - r_j\| - \|\boldsymbol{r}_s - r_k\| = \delta_{jk}$, $j,k = 1,\ldots N$, where N *is* – number of sensors, δ_{jk} – TDOA for pairs of sensors {j,k}, $\mathbf{r}_s = (x, y, z)$ represents the unknown coordinates of the source. These equations have traditionally been solved using iterative numerical techniques such as the Gauss–Newton method, gradient descent, and the Levenberg–Marquardt algorithm [20, 25]. As noted in Inamdar [2] and emphasized in Kravets et al. [26], finding more effective and reliable solution strategies remains an active area of research.

A key advantage of the method introduced in Inamdar [2] is its ability to reduce the TDOA equations to a closed-form system of linear equations. This significantly simplifies the localization process and enables a direct, non-iterative solution.

Unlike many previous studies that validate TDOA techniques using only theoretical assumptions or idealized synthetic signals, our approach incorporates solutions of the 3D acoustic equations and actual noise recordings from a construction site into the model. These recordings are used to simulate realistic wave propagation and sensor response, providing a more accurate assessment of the method's performance in noisy, urban-like environments.

3.1. Acoustic Model

The mathematical model presented below is derived from classical acoustic equations and was previously used in our study [18]. However, for completeness, we briefly outline it here.

The propagation of sound in a three-dimensional homogeneous medium with density ρ, under the assumption of small perturbations from the stationary state, is described in terms of pressure $p(t,x,y,z)$ and particle velocity $u(t,x,y,z)$ at time t and Cartesian coordinates (x,y,z) by the following system of partial differential equations:

$$\frac{\partial \boldsymbol{u}}{\partial t} + \frac{1}{\rho}\nabla p = \sum_{j=1}^{M} f_j(t,x,y,z), \quad \frac{\partial p}{\partial t} + \rho c^2 \nabla \cdot \boldsymbol{u} = 0. \qquad (1)$$

Here, c - the constant speed of sound, p and $\boldsymbol{u}$, respectively, are the changes in pressure and speed of medium points at the vicinity of the steady-state values ρ = const, p_0 = const, and u = 0. The functions $f_j(t,x,y,z)$ describe the acting disturbing sources.

By differentiating the second equation with respect to time and substituting the time derivative of velocity from the first equation, we derive the classical wave equation for pressure:

$$\begin{cases} \frac{\partial^2 p}{\partial t^2} = c^2\left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2}\right) + \sum_{j=1}^{M} S_j(t,x,y,z) \\ p(0,x,y,z) = 0 \end{cases} \qquad (2)$$

In the general case, the solution to the problem (1) is given by a convolution of the fundamental solution of the wave equation with the right-hand side [24]. We consider the case when the characteristic sizes of the sensors and disturbing sources are at least three orders of magnitude less than the scale of the solution area. This makes it possible to model "hot spots" as concentrated pointwise sources. Accordingly, the source intensity function will be written in the form of a generalized δ-function, i.e., Equation 3:

$$S_j(x,y,z,t) = \delta(r - r_j)H_j(t), \ j = \overline{1,M} \qquad (3)$$

The fundamental solution of the wave Equation 2 has the following form [21]:

$$E_3(x,y,z,t) = \frac{1}{4\pi c}\frac{\delta(r-ct)}{r}, \ r = |\mathrm{r}| = \sqrt{x^2+y^2+z^2} \qquad (4)$$

Then, computing the convolution of the function in (3) with the fundamental solution in (4), we obtain the expression for the solution of the Cauchy problem (2) for the source j as (5):

$$p_j(t,r) = \frac{1}{4\pi c}\int_0^{\infty} H_j(t-\tau)d\tau \iint_{|y|=c\tau} \frac{\delta(|y|-c\tau)\delta(|r-y|)}{|y|} dS_{c\tau} = \frac{1}{4\pi c}\frac{H_j(t-|r|/c)}{|r|} \qquad (5)$$

Due to linearity, for several simultaneously acting sources, the solution is written in the form shown in (6):

$$p(t,r) = \frac{1}{4\pi c}\sum_{j=1}^{M} \frac{H_j(t-|r-r_j|/c)}{|r-r_j|} \qquad (6)$$

The signal arrives from each source with a time delay equal to the distance from the detector to the source r_{ij} divided by the wave speed c. When different sensors receive a signal, this difference in arrival time is usually used to derive the system of equations for source localization [16]. On the other hand, the signal arrives with a decay of amplitude reciprocal to the distance from the source. This relationship can also be considered when determining the position of the source.

Equation 6 made it possible to perform numerical modeling of signal propagation using MATLAB scripts and to construct the sound intensity distribution in the solution area. Based on this model, the propagation of a spherical sound wave was modeled, and the detection signals and sources were simulated.

3.2. TDOA-based localization

In this section, we demonstrate the application of the TDOA method, as described in Inamdar [2] for the localization of a signal source and the reconstruction of the emitted signal.

Assume that a signal source is located at point A with Cartesian coordinates (*x*,*y*,*z*). The incoming signal is recorded by detectors positioned at points 1 through 5 (see Figure 1). In this configuration, the dimensions of the detector system are determined by the parameter d (Table 1).

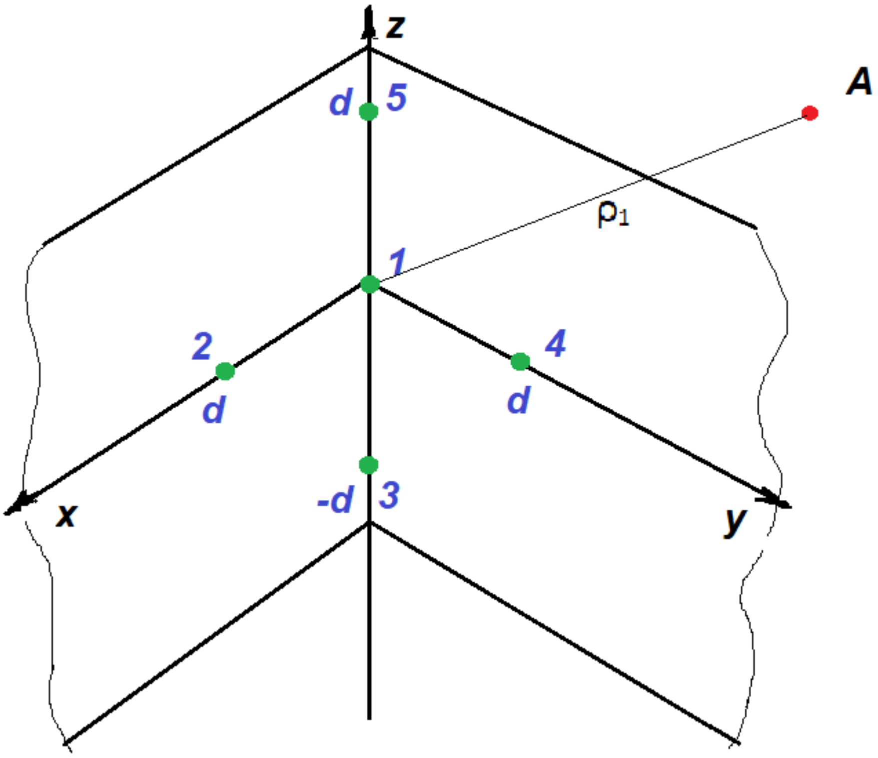

Figure 1.
Schematic representation of the spatial positions of the detectors: *Oxz* and *Oyz* planes correspond to the site fences; *A* – the source; ρ_1 – distance from *A* to the origin placed at sensor No. 1.

Table 1.
Positions of detector system.

Detector No. (*i*)	x_i	y_i	z_i
1	0	0	0
2	d	0	0
3	0	0	-d
4	0	d	0
5	0	0	d

The central point, corresponding to the origin of the coordinate system, is designated as point No. 1. Let ρ_i denote the distance from the *i*-th sensor to the source with Cartesian coordinates (*x*,*y*,*z*):

$$\rho_1 = \sqrt{x^2 + y^2 + z^2};\ \rho_2 = \sqrt{(x-d)^2 + y^2 + z^2};\ \rho_3 = \sqrt{x^2 + y^2 + (z+d)^2};$$
$$\rho_4 = \sqrt{x^2 + (y-d)^2 + z^2};\ \rho_5 = \sqrt{x^2 + y^2 + (z-d)^2} \quad (7)$$

So, we would say that this system of sensors is placed in a 3D domain of size $d \times d \times 2d$.

Note that such a configuration can be implemented in practice, for example, in the corner of a construction site, at the intersection of two fences. In this case, the intersection line of the fences corresponds to the line going along sensors No. 1, No. 3, No. 5, and sensors No. 2, No. 4 are located on the walls of the fence at a distance of d from the corner.

Let $\boldsymbol{r}_i, i = 1,2,3,4,5$ note the radius vectors of the sensors' positions, and $\boldsymbol{r}_s$ = (*x*, *y*, *z*) is the radius vector of the source *A*. Let $\delta_{j1} = \rho_j - \rho_1$ be the difference in distances from the source to the sensor with number j and the sensor with number 1, which has been placed at the origin point. In practice, this distance is determined through the signal delay between sensors 1 and j, multiplied by the speed *c* of signal propagation, in our case by the sound speed in the medium.

In the paper Desai and Mehendale [23], the following system of linear equations has been derived to determine the r_s:

$$Br_s = X, \quad (8)$$

or

$$r_s = B^{-1}X, \quad (9)$$

where the rows of the matrix B are defined in terms of the positions of the sensors:

$$B = 2(r_j - \frac{\delta_{j1}}{\delta_{k1}} r_k). \quad (10)$$

Remark: It follows from formula (10) that if all sensors lie in one plane, then in some coordinate system, all of them have one of the coordinates equal to zero. Then, the matrix $\boldsymbol{B}$ will have a zero column and is not invertible. Therefore, for

the applicability of this method, the sensor system should not be planar. To obtain three rows of matrix ***B***, three different combinations of indices {j,k} must be substituted into the formula (10).
The vector **X** in (8) is defined via the following formula from [2]:

$$\boldsymbol{X} = \left(\delta_{j1}^2 - \delta_{j1}\delta_{k1}\right) + \left(\boldsymbol{r^T}_j\boldsymbol{r_j} - \frac{\delta_{j1}}{\delta_{k1}}\boldsymbol{r}_k^T\boldsymbol{r}_k\right) \qquad (11)$$

Here, to construct three rows of the vector ***X***, we need to select three pairs of indices $\{j, k\}$. The following pairs of indices are possible for our sensor system: {(3,2), (4,2), (4,3), (5,2), (5,3), (5,4)}. To generate the matrix ***B*** and the vector ***X***, it is sufficient to take any three of them. To avoid a possible division by zero in formula (9), when selecting indices, we first multiply Equation 10 by δ_{k1}. Then, substituting the expressions (10) and (11) into the system (8), we obtain the reformulated equations for the position r_s of the source:

$$B'r_s = 2\left(\delta_{k1}\boldsymbol{r}_j - \delta_{j1}\boldsymbol{r}_k\right)\boldsymbol{r_s} = \left(\delta_{k1}\delta_{j1}^2 - \delta_{j1}\delta_{k1}^2\right) + \left(\delta_{k1}\left|\boldsymbol{r_j}\right|^2 - \delta_{j1}|\boldsymbol{r}_k|^2\right) = \boldsymbol{X'} \qquad (12)$$

Further, in our numerical computations, we applied the modified Equations 12 instead of (8).

3.3. Data Collection

The model (2)-(6) contains the functions $H_j(t)$ that describe disturbing signals at source number j at time t. In our case, we need to specify one signal function $H(t)$, which was simulated using recordings of real sounds in an urban environment. We have used the same sounds as in our work [18], specifically, the sounds of some metal cutting tools and working construction machines were recorded on the construction site of the residential complex "Athletic City" located in Astana, Kazakhstan, in July 2023. The sample frequency was 48,000 Hz and recorded by an iPhone using an audio compression technology called Apple Lossless Audio Codec (ALAC). The length of recordings ranged from 5 to 30 seconds, and the distance to the source was relatively close, around 5 m, due to the presence of extraneous noises. In total, we had nine different records, but below, in numerical modeling, we consider only three of them. We applied MATLAB R2023b software to generate 2-second soundtracks as graphs from various noise sources. Although the sounds were originally recorded at a sampling rate of 48,000 Hz, we downsampled the data by retaining every fourth value. Additionally, to bring the sound power to a common scale for modeling purposes, we normalized the values by dividing them by the maximum amplitude within the considered time interval. Figure 2 shows examples of signal graphs with a duration of 22 µs, used in our calculations to generate synthetic data and containing 512 discrete values of the signal amplitude, normalized relative to the maximum amplitude of each signal.

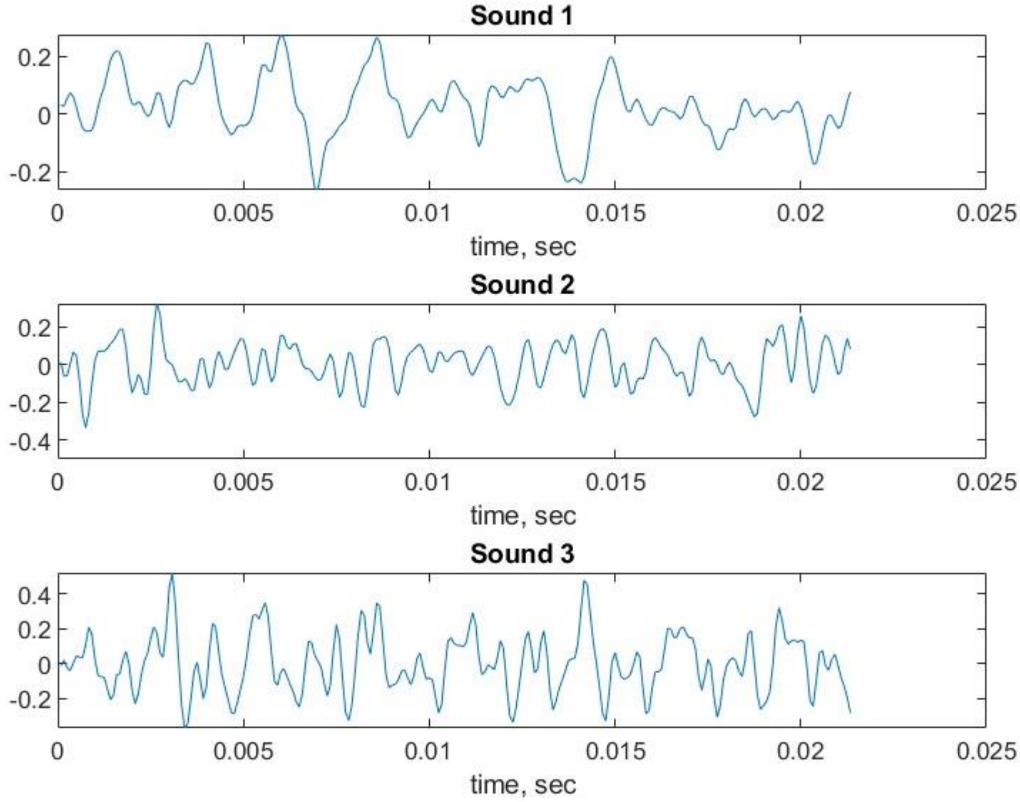

Figure 2.
Examples of sound signals (512 samples are presented).

In addition to real signals recorded in urban environments, at the preliminary stage of modeling, we also attempted to use simple signals containing 1-2 harmonic oscillation frequencies. In this case, the MATLAB function *align signals* produced large errors in time-delay estimation. Accurate delay estimation requires the signal to have sufficient spectral diversity within the recorded duration of N values.

4. Results and Discussion

To ensure numerical stability and maintain computational efficiency in our sound propagation simulations, we applied a system of scaled units adapted to the physical characteristics of the problem. This scaling approach facilitates the handling of numerical data within an optimal computational range. Specifically, we defined the characteristic scales as follows:

- Length scale: L = 10 meters
- Time scale: T= 0.01 seconds
- Sampling frequency: 120 samples per T unit, corresponding to 12000 Hz
- Speed scale: 100 meters per second
- Normalized speed of sound: c=0.34 in scaled units.

By adopting these scales, all subsequent computations are performed within a normalized framework, enhancing numerical precision and facilitating the interpretation of simulation results.

We performed numerical calculations for the following pairs of indices to generate a matrix $\boldsymbol{B'}$: {(3,2),(4.3),(5,4)}. We write out the explicit form of matrix $\boldsymbol{B}$ in this case:

$$\boldsymbol{B'} = 2d\begin{bmatrix} \delta_{21}-\delta_{31} & 1 & \delta_{21}-\delta_{31} \\ \delta_{31}-\delta_{41} & \delta_{31}-\delta_{41} & 0 \\ -\delta_{41} & -(\delta_{41}+\delta_{51}) & 0 \end{bmatrix} \quad (13)$$

Vector $\boldsymbol{X}$ in this case has the form:

$$\boldsymbol{X'} = \begin{bmatrix} (\delta_{21}\delta_{31}^2-\delta_{31}\delta_{21}^2)+2d^2(\delta_{21}-\delta_{31}) \\ (\delta_{31}\delta_{41}^2-\delta_{41}\delta_{31}^2)+2d^2(\delta_{31}-\delta_{41}) \\ (\delta_{41}\delta_{51}^2-\delta_{51}\delta_{41}^2)+2d^2(\delta_{41}-\delta_{51}) \end{bmatrix} \quad (14)$$

From the structure of the matrix (13), the importance of the quantities δ_{j1} becomes obvious. For the matrix $\boldsymbol{B'}$ to be invertible, in addition to the requirement that the sensors do not all lie in one plane, we must avoid combinations where $\delta_{21}=\delta_{31}$. This case needs to be considered separately or to take other pairs of the possible set of indices {j, k}: {(3,2), (4,2), (4,3), (5,2), (5,3), (5,4)}, where the rank of the matrix $\boldsymbol{B'}$ equals 3.

Thus, we have solved the following system of equations:

$$\boldsymbol{B'}\boldsymbol{r}_s = \boldsymbol{X'} \quad (15)$$

with matrices defined in Formulas (13) and (14).
Table 2 presents the results obtained for different source positions.

Table 2.
Results of recovery of a single source via the system of detector of 5 sensors in the coordinate system related to the reference point No. 1 (Figure 1).

No.	d, m	Number of discrete samples, *N*	Exact position of the source			Noise-free computation of source position			Recovered position of source via alignment of discrete signals			Exact distance to the source, ρ_1, m	Recovered distance to the source, $\rho_{1,recov}$, m	Relative error, % $\frac{\lvert\Delta\rho_1\rvert}{\rho_1}$
			x, m	*y*, m	*z*, m	*x*, m	*y*, m	*z*, m	*x*, m	*y*, m	*z*, m			
1	1	256	8	9	-10	8	9	-10	3.74	4.10	-4.47	15.65	7.12	54%
2	1	1024	8	9	-10	8	9	-10	3.74	4.10	-4.47	15.65	7.12	54%
3	2	512	8	9	-10	8	9	-10	7.11	8.02	-8.93	15.65	13.95	11%
4	1	512	2	1	10	2	1	10	0.22	0.41	-1.54	10.25	1.61	84%
5	2	512	3	4	2	3	4	2	3.01	4.01	2.04	5.39	5.41	1%
6	2	512	4	4	2	4	4	2	3.95	3.95	1.97	6.00	5.92	1%
7	2	512	4	5	2	4	5	2	4.18	5.27	2.08	6.71	7.04	5%
8	2	512	4	5	1	4	5	1	4.02	4.98	1.00	6.48	6.47	0%
9	2	512	6	5	1	6	5	1	6.12	5.15	1.00	7.87	8.06	2%
10	2	512	6	8	1	6	8	1	6.32	8.45	1.00	10.05	10.60	5%
11	2	512	6	8	-2	6	8	-2	6.16	8.10	-2.02	10.20	10.37	2%
12	2	256	7	8	-10	7	8	-10	6.18	7.01	-8.63	14.59	12.72	13%
13	2	512	7	8	-10	7	8	-10	6.18	7.01	-8.63	14.59	12.72	13%
14	2	1024	7	8	-10	7	8	-10	6.18	7.01	-8.63	14.59	12.72	13%
15	2	256	8	8	-10	8	8	-10	8.45	8.45	-10.55	15.10	15.94	6%
16	2	512	6	8	-10	6	8	-10	5.94	8.04	-10.09	14.14	14.20	0%
17	2	512	6	8	-12	6	8	-12	5.65	7.42	-11.22	15.62	14.59	7%
18	1	512	6	8	-12	6	8	-12	2.93	3.90	-5.61	15.62	7.43	52%

In these numerical experiments, the base distance d and the number of samples N used for calculating the time delay between signals were varied. Here, in addition to the exact coordinates of the source, we also present coordinates calculated from synthetic data defined by (7) without accounting for discretization (i.e., noise-free computation); in all cases, the relative error was below 0.01%. Furthermore, the delays δ_{j1} for j = 2,3,4 were computed for real discrete signals using the MATLAB function *alignsignals*. Since the signals are represented by discrete samples with a fixed step size, an error in the calculation of δ_{j1} is inevitable. This discretization error was found to significantly impact the overall accuracy of the position calculations. The final column of the table reports the relative error in calculating the distance from the source to the origin. The red-colored lines indicate cases where the method failed to detect the source position correctly. Numerical computations show that for sources placed at distances approximately 10 times greater than the value d, errors in computing time delays lead to discrepancies in source position calculations.

The three columns "Recovered position of…" in Table 2 present the reconstructed coordinates of the source, derived from the analysis of discrete signals. It was observed that increasing the base distance d enhances the accuracy of the coordinate calculations (lines 17, 18, and 2, 3). If the number N is sufficiently large to allow the signal lag to be calculated correctly from these data, a further increase in N almost does not affect the accuracy of the calculations (lines 1, 2, and 12-14). This effect is especially noticeable if the source is located at a significant distance from the center, approximately an order of magnitude greater than the size d. If the source is placed at distances comparable to the value d, the results give satisfactory accuracy (lines 5-11). The most important factor was the rank of the matrix $\boldsymbol{B}$. Line 4 represents a case where the matrix rank is 2 (less than 3), which requires separate analysis.

In the condition of urban sites, the admissible range for d was found to be between 1 m and 2 m, with N values ranging from 256 to 1024, corresponding to a detectable signal duration of approximately 11 to 44 µs. Utilizing significantly larger N values increases computational load and was deemed inefficient because it did not affect the accuracy.

After determining the distance to the signal source, one can estimate the attenuation coefficient, which facilitates the reconstruction of the original signal amplitude at the source. Assuming A_0 represents the amplitude of the signal in the immediate vicinity of the source, the amplitudes A_1 and A_2 measured at sensors located at distances ρ_1 and ρ_2 from the source can be expressed as:

$$A_1 = \frac{A_0\rho_0}{\rho_1};\ A_2 = \frac{A_0\rho_0}{\rho_2} \tag{16}$$

Here, ρ_0 is a reference distance sufficiently close to the source, chosen based on practical considerations. For instance, in a construction site scenario, where tools generate sound within a region approximately 0.1 meters in size, and considering a scaling factor L=10 meters, one might set ρ_0=0.01.

Given that the source coordinates and the distance ρ_1 to the first sensor have been previously determined, and knowing the coordinates of a second sensor to calculate ρ_2, we derive the following relationships:

$$A_1 = A_2\frac{\rho_2}{\rho_1};\quad A_0 = A_1\,\rho_1/\rho_0 \tag{17}$$

These equations imply that the amplitude A_0 in the vicinity of the source can be estimated by scaling the measured amplitude A_1 at a detector by the ratio ρ_1/ρ_0. This approach aligns with the inverse distance law for sound pressure, which states that sound pressure decreases proportionally to the inverse of the distance from the source. The knowledge of that amplitude is important to monitor the level of noise produced in that urban area.

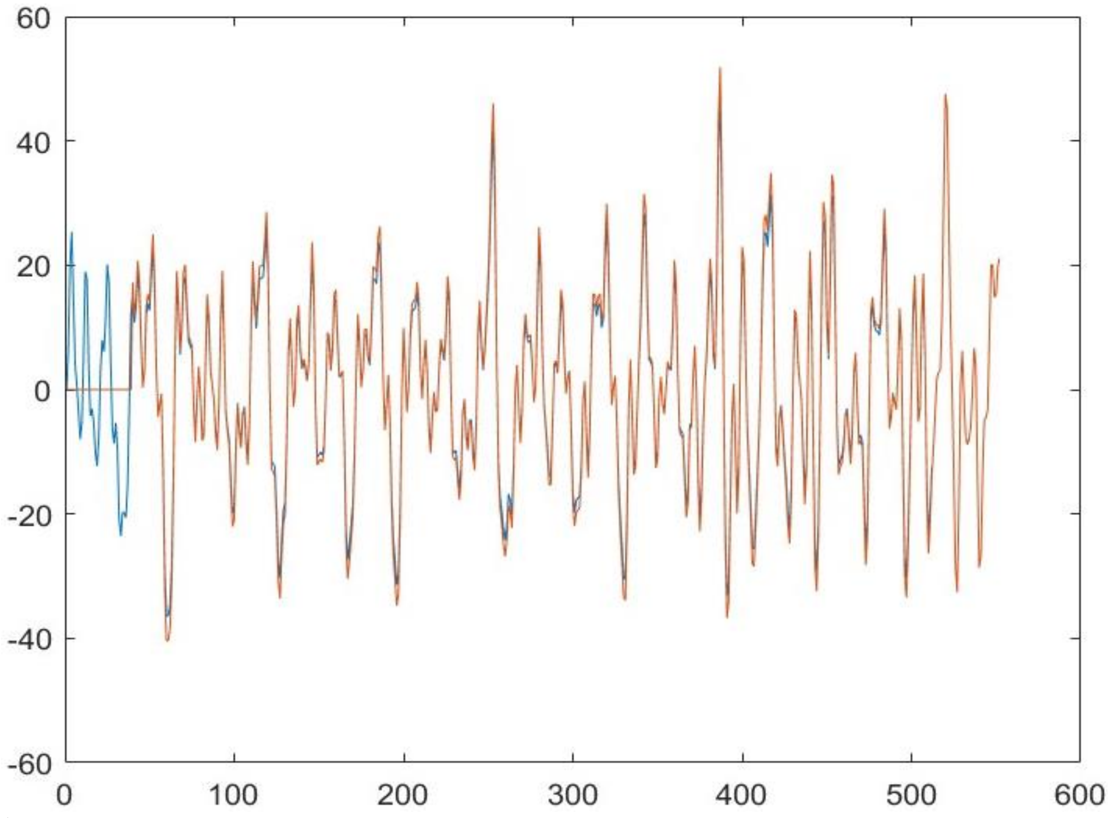

Figure 3.
Recovered signal (red line) aligned along the original signal (blue line) at sensor No. 1.

Figure 3 compares the original signal with the reconstructed signal after time alignment, accounting for relative delay. It can be seen that the quality of the recovered amplitude is satisfactory.

5. Discussion

As shown in the "Noise-free computation" column of Table 2, our results are in full agreement with those reported in Inamdar [2], where the author also demonstrated high localization accuracy, provided that the system matrix is non-degenerate. However, inevitable errors in time-delay estimation significantly affect the reliability of the method.

The present findings are consistent with recent advances in TDOA-based localization research [28, 29]. Our observation that optimal performance is achieved when sensor spacing is comparable to the distance to the source supports conclusions from studies on sensor placement, which recommend star-shaped or spatially well-distributed array geometries for improved accuracy [28].

Classical TDOA methods have been criticized for high error variance due to differencing operations and sensitivity to noise amplification [30]. In contrast, our approach, which solves a closed-form linear system, avoids the convergence issues associated with iterative methods. Nonetheless, time-delay estimation remains the main source of error, as also observed in our results. Furthermore, under non-line-of-sight and reverberant conditions, TDOA methods tend to produce biased estimates unless advanced compensation techniques are applied [31], highlighting the need for careful sensor deployment and algorithmic refinement in future studies.

6. Conclusion

In this work, we applied a mathematical model for acoustic wave propagation to the problem of sound source localization in urban environments. Linear closed-form equations based on the TDOA method were used to estimate the coordinates of a sound source using a 3D array of detectors. The method demonstrated high accuracy in noise-free conditions, aligning well with previous studies. This modeling strategy aims to support a more practical evaluation of localization accuracy and may also be useful in preparing datasets for future machine learning applications. By including real noise in the simulation process, this study attempted to reduce the gap between theoretical analysis and practical deployment and to better understand the limitations and applicability of the method in realistic settings.

6.1. Limitations

The primary source of localization error identified in this study is the imprecision in estimating the time delays between signals detected by different sensors. This issue becomes more pronounced when the time resolution is insufficient. While increasing the base distance between detectors and the sampling step can mitigate this error, doing so also requires a proportional increase in the number of signal samples to maintain alignment accuracy. Additionally, the scenario where the source is positioned such that the rank of the localization matrix B is less than 3 remains unaddressed and poses a challenge for future investigation.

6.2. Implications

The presented mathematical model enables flexible numerical simulation with an arbitrary number of sources and various detector configurations. This versatility makes the approach valuable for future applications, particularly in the generation of synthetic data for training neural networks in machine learning tasks. The findings also offer practical guidance for optimizing microphone array configurations to improve the accuracy of sound source localization in real-time acoustic monitoring systems, especially in complex urban or industrial environments.

6.3. Future Work

Future research should focus on exploring degenerate configurations of detectors and sources, particularly where the system matrix lacks full rank. Another important direction is the extension of the model to handle multiple simultaneous sources using expanded detector networks or adaptive algorithms. Moreover, integrating the method with real-time signal processing systems and testing it in dynamic environments will further validate its practical potential. Applying the method as a simulation tool to generate labeled datasets for supervised learning in source localization tasks also holds significant promise.

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